UNSTEADY CONVECTIVE HEAT TRANSFER IN NATURAL COOLING OF VERTICAL PLATES

I. V. Il'inskii, É. E. Prokhach, and V. P. Pershin

UDC 536.25

The results of an experimental investigation of the heat transfer of a vertical thin plate (Bi $\ll 0.1$) in an undisturbed infinite medium are given.

Before the experiment the plate was heated to 100-200°C by a brief (0.4-1 sec) passage of electric current. The heat transfer of the hot plate was investigated as it cooled naturally in air, water, kerosene, and transformer oil. The temperatures of the plate and medium were measured and the optical picture of the spectrum of convection currents was photographed. The heat transfer coefficients were determined from the change in heat content of the plate, and also by the regular heat regime method on some parts of the temperature curve.

The initial values of the Rayleigh number in the experiments in air were $\sim 1.4 \cdot 10^7$, which corresponded to an unstable region near the boundaries of thermal turbulence. The brief heating led to rapid swelling of the boundary layer and the slow development of curling motion. We found that at a cooling rate of about 1 deg/sec the process became stabilized (quasistationary), and values of Nu were given satisfactorily by the known formula

$$\mathrm{Nu} = 0.135(\mathrm{GrPr})^{\frac{1}{3}}.$$

This is illustrated by Fig. 1. As the dimensionless time we used the product FoBi = $m\tau$, where m is the cooling rate, and τ is the time.

As distinct from experiments in air, the experimental data for drop-forming liquids did not correspond with the relationships obtained for steady conditions. Similarity theory and physical modeling techniques were used to obtain a general relationship for the heat transfer during cooling of a plate in an infinite medium:

$$\mathrm{Nu}_x = 0.44\left(\frac{\mathrm{Fo}}{Q}\right)^{-0.5}(\mathrm{Gr}_0\mathrm{Pr}_0^{0.5})^{0.13}.$$

The following ranges of numbers were used in the experiments

$$\mathrm{Gr} = 5\cdot 10^{-6} - 2\cdot 10^{9},\ \mathrm{Pr} = 0.72 - 200,\ \mathrm{Fo} = 9\cdot 10^{-6} - 2.6\cdot 10^{-1},$$

$$Q = \frac{c_w\gamma_w}{c_f\gamma_f}\cdot\frac{\delta}{l} = 1.05\cdot 10^{-3} - 4.5,$$

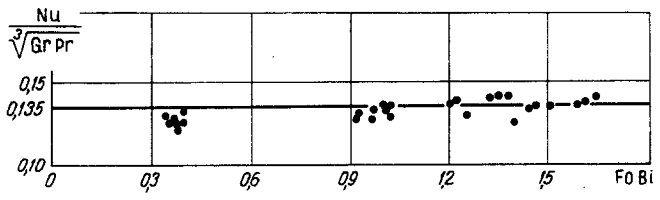

Fig. 1

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 27, No. 3, pp. 524-539, September, 1974. Original article submitted May 3, 1972.

where c is the specific heat, γ is the specific weight, δ is the plate thickness, l is the distance from the end face of the plate to the point of temperature measurement.

The subscript ω indicates plate parameters, f indicates parameters of the medium, and 0 the initial values of the parameters.

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Original article submitted May 3, 1972.

EFFECT OF AERODYNAMIC CHARACTERISTICS OF AN INSTRUMENT ON THE DRIVING FORCE OF HEAT AND MASS TRANSFER

Yu. I. Volovik

UDC 66.047.3

The processes of heat exchange and mass exchange between water and hot air in an atomizing chamber are examined for the cases where the chamber represents an instrument of ideal displacement and one of ideal mixing.

With the choice of the following variable dimensionless complexes

$$\Theta = \frac{t_0 - t_c}{t_0 - t_w}; \quad A = \frac{\alpha F}{G_a c_a}; \quad K = \frac{r}{c_s (t_0 - t_w)};$$

$$\Phi = \frac{P_c - P_0}{P_s - P_0}; \quad B = \frac{\beta F M_a (P - P_0)(P - P_s)}{G_a M_v P}; \quad N = \frac{P - P_s}{P_s - P_0}$$

we obtain the dependences $\Theta = f(A, K)$ for the heat exchange and $\Phi = f(B, N)$ for the mass exchange which have identical forms, since the results obtained in an examination of heat exchange are also valid for mass exchange with the appropriate substitution of Θ, B, and N for Φ, A, and K.

Instruments for which the values of A and K are identical can be compared with respect to the complex Θ, called the efficiency of the instrument in the present report. In a comparison of the values of Θ for chambers of ideal displacement and of ideal mixing in the range of values $0 \le A \le \infty$ and $5 \le K \le \infty$ we determine that the greatest absolute difference $\Theta_d - \Theta_m$ is about 0.2 (see curve 2 of Fig. 1).

The graphic dependence $\Theta_d = f(\Theta_m)$, which is valid in the range of $5 \le K \le \infty$, is presented in Fig. 1. The equation $\Theta_r = \Theta_m + \Delta\Theta$, where $\Delta\Theta$ is determined from Fig. 1 (curve 3), can be recommended for the determination of Θ_r for industrial instruments. In this case the error in determining the instrument efficiency will not exceed ±13%. By expressing the instrument efficiency through the transfer potential Δt_{av} averaged over the process we find

$$0.75 < \frac{\Delta t_{av\,m}}{\Delta t_{av\,d}} < 1.$$

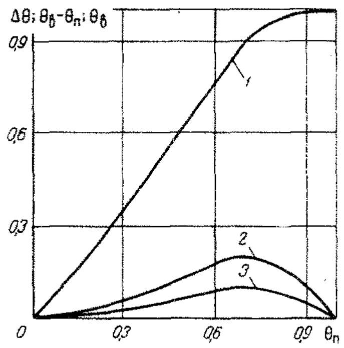

Fig. 1. Relationship between efficiencies of instruments of ideal mixing and of ideal displacement: 1) $\Theta_d = f(\Theta_m)$; 2) $\Theta_d - \Theta_m = f(\Theta_m)$; 3) $\Delta\Theta = f(\Theta_m)$ for the case where the aerodynamic characteristics of the instrument are unknown.

NOTATION

α, heat exchange coefficient; β, mass exchange coefficient; F, surface of heat and mass exchange; G_a, flow rate of air; t_0, initial air temperature; t_c, air temperature at exit from chamber; t_w, water temperature; c_a, heat capacity of air; c_v, heat capacity of water vapors; r, latent heat of evaporation; P_s, partial pressure of water vapor at surface of drop; P_c, partial pressure of water vapor in air at exit from chamber; P_0, partial pressure of water vapor in air at entrance to chamber; P, total pressure in chamber; M_a, molecular weight of air; M_v, molecular weight of vapor. Subscripts: d, m, and r refer to instruments of ideal displacement and of ideal mixing and to real instruments of intermediate type, respectively.

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AEROSOL OF VARIABLE MASS IN A VORTEX FLOW

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UDC 532.529.5

Let us write the equation of motion of a particle of variable mass. Inertial forces, centrifugal forces, and forces of aerodynamic resistance act on the particle. The swirling of the flow is accomplished by a vane swirler. Thus:

$$\frac{d}{dt}(m_a V_r) - \frac{1}{12}\pi d^2 \rho \frac{dV_r}{dt} = m_a \frac{\theta V^2}{r} - m(k+1)\frac{V^2}{r} - C_x \frac{\rho (V_r - V_R)^2}{8}\pi d^2, \tag{1}$$

where $m_a = 1/6\ \pi d^3 \rho_a$; $m = 1/6\ \pi d^3 \rho$; $d = d_0\ t/\tau$; θ is the coefficient of slip of the particle relative to the tangential velocity of the flow; $k \approx 12$; d_0 is the particle diameter; τ is a proportionality constant; ρ is the mass density of the gas, ρ_a is the mass density of the aerosol; r is the radial coordinate directed from the axis of rotation of the flow toward the periphery of the apparatus of radius R; t is the time; V_R is the radial component of the velocity of the gas flow; V is the tangential component of the gas flow; C_x is the coefficient of frontal resistance; V_r is the unknown velocity of the particle under consideration. The solution of Eq. (1) has the form

$$V_r(t) = \frac{4\left(1 - \frac{\rho}{\rho_a}\right)\rho_a d_0}{3C_x \rho t}\{[C_1 J_p(\beta t^\gamma) + C_2 N_p(\beta t^\gamma)]\alpha + (\gamma - 1)\beta t^\gamma [C_1 J_{p-1}(\beta t^\gamma) + C_2 N_{p-1}(\beta t^\gamma)]\}, \tag{2}$$

where $C_1 = a/b - a$, $C_2 = 1/b - a$, $a = -N_p + N_{p-1} + N_{p-2}/-J_p + J_{p-1} + J_{p-2}$, $b = (N_p - N_{p-1})\alpha$, and J_p and N_p are Bessel and Neumann functions, respectively;

$$\beta t^\gamma = \sqrt{\frac{3C_x\left[\theta - \frac{\rho}{\rho_a}(k+1)\right]V^2 \rho \tau t}{d_0 \rho_a r\left(1 - \frac{\rho}{\rho_a}\right)}};$$

$$\gamma = \frac{1}{2}; \quad \beta = \sqrt{\frac{3C_x\left[\theta - \frac{\rho}{\rho_a}(k+1)\right]V^2 \rho \tau}{d_0 \rho_a r\left(1 - \frac{\rho}{\rho_a}\right)}};$$

$$p = \frac{9}{\left(1 - \frac{\rho}{\rho_a}\right)^2}\left[1 - \frac{1}{2}A - \frac{3}{16}A^2\right];$$

$$A = \frac{C_x V_R \tau \rho}{d_0 \rho_a}; \quad \alpha = \frac{3}{2\left(1 - \frac{\rho}{\rho_a}\right)}\left(\frac{1}{4}A - 1\right);$$

$$\frac{V^2}{r} = \varphi\sqrt{\frac{2H}{\rho}} \cdot \frac{\cos\beta_0}{r} = \text{const};$$

H is the pressure drop at the blade swirler and β_0 is the mounting angle of the swirler blades. V_R and V are determined quantitatively by a method presented in JEP, 18, No. 3 (1970).

With d = const

$$V_R = \frac{\left[-\frac{\gamma_2}{\gamma_1-\gamma_2}\gamma_1 \exp(-\gamma_1)t + \frac{\gamma_1}{\gamma_1-\gamma_2}\gamma_2 \exp(-\gamma_2)t\right]}{\left[-\frac{\gamma_2}{\gamma_1-\gamma_2}\exp(-\gamma_1)t + \frac{\gamma_1}{\gamma_1-\gamma_2}\exp(-\gamma_2)t\right]B}; \tag{3}$$

$$\gamma_1 = \frac{BV_R-\lambda}{2}; \quad \gamma_2 = \frac{BV_R+\lambda}{2}; \quad \lambda = \frac{1}{2}\sqrt{(BV_R)^2+4D};$$

$$B = \frac{1}{1-\frac{\rho}{\rho_a}} \cdot \frac{3}{4} C_x \frac{\rho}{\rho_a} \cdot \frac{1}{d};$$

$$D = B\left\{\frac{1}{1-\frac{\rho}{\rho_a}}\left[\theta - \frac{\rho}{\rho_a}(k+1)\right]\frac{V^2}{r} - BV_R^2\right\}.$$

Equations (2) and (3) were confirmed experimentally on a vortex dust-collecting apparatus (R = 100 mm; $d_0 = 5\ \mu$; $\beta_0 = 20°$; H = 210 mm water column; k = 11.5; $\theta = 0.85$; $\varphi = 0.91$; $C_X = 30$–14; $\tau = 0.04$–0.06).

Equations (2) and (3) can be used in constructing a method of calculating vortex instruments.

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Original article submitted June 14, 1972

STUDY OF THE POSSIBILITY OF USING EMPIRICAL FUNCTIONS OF THE COEFFICIENT OF RESISTANCE FOR THE CALCULATION OF CURVILINEAR DUST-LADEN FLOWS

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UDC 532.529.5

In the study of the working processes in various instruments which have a two-phase flow (gas-dust, gas-liquid drops) a calculated determination is very often made of the particle trajectories and velocities. In this case the differential equations of motion involve the coefficient of frontal resistance C which is a function of the Reynolds number of the particle (the dimensionless velocity of its blowing): $Re = u\delta\rho_1/\eta$ (δ and u are the diameter and relative velocity of the particle; ρ_1 and η are the density and dynamic viscosity of the carrying medium).

Since the function C(Re) has a very complicated nature and it is difficult to approximate it accurately enough in a wide range of Re a great many interpolation equations $C = f(Re)$ have been proposed which permit one to calculate C in a given range of variation of Re with one or another accuracy. When using an electronic computer for the calculation it is most advisable to determine C from a table of $C = f(Re)$ compiled on the basis of the most reliable experimental data generalizing the studies of many authors. However, the use of various empirical equations for C continues in the earlier works of many authors at present, and therefore it is advisable to estimate the error produced by them, particularly in light of the fact that the error in the determination of C can itself give little indication of the size of the error in the determination of the trajectory since other factors besides the force of resistance have a large effect.

Calculations are made of the trajectories of particles 16.5–427 μ in diameter in a curvilinear annular air flow 1000 mm in diameter with a determination of the relative separation angle (the angular path up to contact with the outer wall). Trajectories obtained using an experimental function C(Re) are taken as the standard. Fourteen interpolation equations of different authors are tested and the relative error Δ

in the determination of the separation angle is found for each one. In this case the air speed and the initial velocity of insertion of the particles into the channel were varied. The principal results of the study are:

1. The use of a majority of the more complicated many-termed functions C(Re) leads to an error $\Delta \le 7\%$.

2. The use of simple functions of the type $C = 0.4$, $C = 10/\sqrt{Re}$, etc. gives an error of 40-75%.

3. The most successful interpolation equations are the following: $C = 24/Re + 4\sqrt[3]{Re}$, $24/Re + 3.6/Re^{0.313}$, $24/Re + 2.8/\sqrt[4]{Re}$, which lead to errors not exceeding 5-7% in the ranges of the variables studied.

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V. I. Lenin Ivanovo Power Institute
Original article submitted June 22, 1972

CONCENTRATION OF SUSPENDED PARTICLES OF VARIABLE MASS IN A TURBULENT FLOW

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UDC 532.529.5

The problem of the concentration of a hygroscopic aerosol in a confined turbulent air flow behind a source of constant intensity reduces to the solution of the turbulent diffusion equation

$$\frac{\partial u_x C}{\partial x} + \frac{\partial u_z C}{\partial z} = D_T \frac{\partial^2 C}{\partial z^2} \tag{1}$$

with boundary conditions

$$D_T \frac{\partial C}{\partial z} - u_z C\Big|_{z=1} = 0,$$

$$C|_{z=0} = 0,$$

where

$$u_x = \text{const}, \quad u_z = aR^2. \tag{2}$$

The change in the radius of hygroscopic particles is due to growth by consideration and coagulation

$$\frac{\partial R}{\partial z} = \frac{A}{aR^6} + BR, \tag{3}$$

where

$$A = \frac{D\mu \Delta P_0 R_0^3}{\rho R_T T}, \qquad B = \frac{1.37\Delta\omega\varepsilon^{\frac{3}{4}}}{\rho_0 a \nu^{\frac{5}{4}}}.$$

Determining R from Eq. (3) we obtain

$$u_z = a\left[R_0^7 + \left(\frac{7A}{a} + 7BR_0^7\right)(h - z)\right]^{\frac{2}{7}}. \tag{4}$$

The solution of Eq. (1), taken in conjunction with (2) and (4), will be sought in the form

$$C(x, z) = X(x) Z(z).$$

Then, using the Ritz method, we obtain a first approximation to the mean cross-sectional concentration of the aerosol

$$C_m \cong \exp\left(-\frac{\lambda}{Pe} x\right). \tag{5}$$

The eigenvalue λ depends on the initial parameters of the hygroscopic aerosol and the air flow.

The obtained solution (5) was tested in the conditions of the potash mines of the "Beloruskalii" combine. A comparison of experimental measurements of the particle concentration with the results of calculation showed good agreement.

The obtained solution provides a basis for the selection of the climatic parameters of air flows which will ensure minimum dust pollution of the atmosphere in potash and salt mines.

NOTATION

D_T, turbulent diffusion coefficient; u_x, averaged flow velocity; u_z, particle settling velocity; R_0, initial particle radius; ρ, ρ_0, density of particles and air, respectively.

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Original article submitted September 7, 1972.

MODEL EQUATIONS OF THE KINETICS OF DESORPTION

L. K. Tsabek

UDC 541.183

The equilibrium kinetics of desorption in porous symmetrical grains is described by a quasilinear parabolic material balance equation and the equation of an isotherm on the phase boundary

$$\frac{\partial (c+q)}{\partial t} = \frac{\partial^2 c}{\partial r^2} + \frac{\nu}{r}\frac{\partial c}{\partial r}, \quad q = f(c). \tag{1}$$

An accurate numerical solution of the partial-differential equation (1) leads to an accurate ordinary differential equation which describes the kinetics of desorption in the outer diffusion region

$$\frac{\partial \bar{q}}{\partial t} = \gamma_i \omega(\bar{q})[\bar{c} - \varphi(\bar{q})], \quad \varphi^{-1} = f, \quad \bar{q} = (1+\nu)\int_0^1 q(c) r^\nu dr. \tag{2}$$

The kinetics of desorption in a mixed region (outer and inner diffusion) is described by the equation

$$\frac{\partial \bar{q}}{\partial t} = \left[\frac{1}{\gamma_i \omega(\bar{q})} + \frac{1}{\gamma_e}\right]^{-1}[\bar{c} - \varphi(\bar{q})]. \tag{3}$$

We find the analytic relation $\omega(\bar{q})$ in the form

$$\omega(\bar{q}) = \frac{1}{(a_6 - \bar{q} + \delta_0)^2}[b_0 + b_1(\bar{q} - \delta_0) + b_2(\bar{q} - \delta_0)^2 + b_3(\bar{q} - \delta_0)^3], \tag{4}$$

The value of δ_0 is chosen from the conditions of max ω and a_6 = max $\bar{q}$. For practical calculations of the kinetics of desorption with $a_6 = 1$, max $\omega = 2.5$, the Langmuir isotherm is $q = (1 + p)c/(1 + pc)$. Figure 1

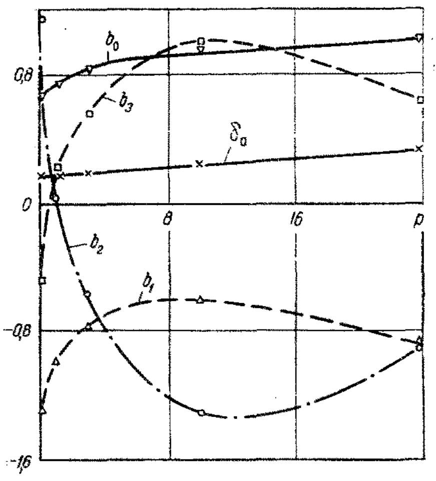

Fig. 1. Values of the coefficients b_i (notation as in Fig. 2).

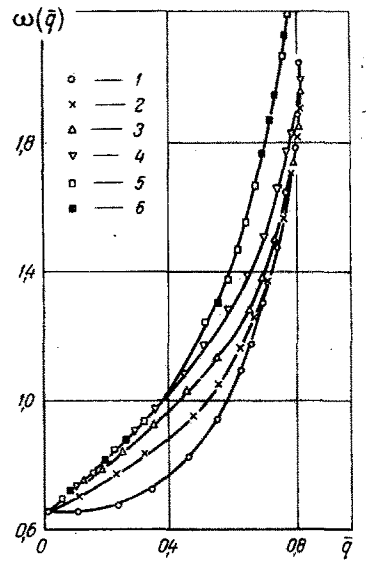

Fig. 2. $\omega(\bar{\bar{q}})$: 1) p = 0; 2) 1; 3) 4; 4) 10; 5) 24; 6) 49.

gives the values of the coefficients b_i for various values of p, and Figure 2 shows graphs of the functions $\omega(\bar{q})$. Using the values of b_i or $\omega(\bar{q})$, the kinetic desorption curves in the inner and mixed regions can be calculated from Eqs. (2) and (3) for arbitrary parameters p.

Dep. 737-74, November 21, 1972
Original article submitted August 2, 1972.

ON A HEAT-TRANSFER PROBLEM

M. P. Lenyuk

UDC 517.944(07)

Two one-sided bounded bodies with different physical parameters and initial data characterizing their initial state (initial temperature distributions) are brought into contact.

Taking account of the finite rate of propagation of heat, the problem of determining the temperature distributions at any instant in the contacting bodies and the heat flux at their boundary is formulated mathematically as follows: it is required to find the bounded solution of the equations [1, 2]

$$b_1^2 \frac{\partial^2 u_1}{\partial t^2} + c_1^2 \frac{\partial u_1}{\partial t} - a_1^2 \frac{\partial^2 u_1}{\partial x^2} = f_1(t, x) \quad (0 \leqslant t \leqslant T,\ 0 \leqslant x < \infty), \tag{1}$$

$$b_2^2 \frac{\partial^2 u_2}{\partial t^2} + c_2^2 \frac{\partial u_2}{\partial t} - a_2^2 \frac{\partial^2 u_2}{\partial x^2} = f_2(t, x) \quad (0 \leqslant t \leqslant T,\ -\infty < x \leqslant 0), \tag{2}$$

which satisfies the conditions

$$u_1|_{t=0} = \varphi_1(x), \quad \left.\frac{\partial u_1}{\partial t}\right|_{t=0} = \psi_1(x) \quad (x \geqslant 0), \tag{3}$$

$$u_2|_{t=0} = \varphi_2(x), \quad \left.\frac{\partial u_2}{\partial t}\right|_{t=0} = \psi_2(x) \quad (x \leqslant 0), \tag{4}$$

$$u_1(+0, t) = u_2(-0, t), \quad \lambda_1 \left.\frac{\partial u_1}{\partial x}\right|_{x=0} = \lambda_2 \left.\frac{\partial u_2}{\partial x}\right|_{x=0}. \tag{5}$$

By introducing Cauchy and Green's functions and the fundamental function of the problem posed above, formulas are obtained describing the required quantities and making it possible to obtain solutions for a broad class of initial data. In the limit as $b_1 \to 0$, $b_2 \to 0$ the temperature distributions are obtained if the rate of propagation of heat is infinitely large in the media $x \geq 0$ or $x \leq 0$ or both simultaneously.

If the rate of propagation of heat in both bodies is infinitely large and there are no heat sources, we have Tsoi's results [1].

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Original article submitted September 26, 1972.

METHOD OF CALCULATING THERMODYNAMIC PROPERTIES OF n-ALKANES FROM ULTRASONIC MEASUREMENTS AT PRESSURES UP TO 2000 ATM

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UDC 536.7

The calculation of the thermodynamic properties of liquids from the results of measurements of sound velocity in relation to pressure and density can be based on the well-known expression for sound velocity, rewritten in the form

$$\gamma=\frac{\left[\frac{\partial (C^3)}{\partial P}\right]_T}{3\left(\frac{\partial C}{\partial \rho}\right)_T}. \tag{1}$$

The literature contains the following empirical relations for the determination of the derivatives in expression (1) for a relatively narrow range of pressures:

$$C^3=C_0^3+K(P-P_0), \tag{2}$$

$$C=C_0+B(\rho-\rho_0), \tag{3}$$

where C_0 and ρ_0 are the sound velocity and density at atmospheric pressure.

As follows from (1), relationships (2) and (3) presume that γ is independent of pressure on the isotherms:

$$\gamma=\frac{K}{3B}. \tag{4}$$

Sound velocity measurements made by the authors for several n-paraffins in the pressure range 0-2000 atm and temperature range 30-120°C indicated that the experimental data deviated from relationships of the form (2) and (3). This indicates that calculation from (2) and (3) should be regarded as calculation based on the mean value of γ for the pressure range in which the results of measurements are being processed.

The authors propose the following expressions for the derivatives in (1):

$$C=\sqrt[3]{(C_0+Z_1)^3+K_0(P-P_0)}-Z_1; \tag{5}$$

$$\rho=\rho_0+\frac{3}{K_0}\left[(C-C_0)+2(Z_1+Z_2)\ln\frac{C-Z_2}{C_0-Z_2}-(Z_1+Z_2)^2\left(\frac{1}{C-Z_2}-\frac{1}{C_0-Z_2}\right)\right], \tag{6}$$

where K_0 is a constant for the whole series of n-paraffins; Z_1 and Z_2 are constants for a particular isotherm and characterize the shift along the velocity axis on superposition of the isotherms on the C-P and C-ρ axes.

Expressions (5) and (6) presume that γ decreases with pressure

$$\gamma=\frac{C^2}{(C-Z_2)^2}. \tag{7}$$

Relationship (5) was obtained by applying similarity theory to the treatment of the experimental data for the sound velocity in the series of n-paraffins in relation to pressure. Expression (6) was obtained from (1) and (5) on the basic assumption that

$$\lim_{p\to\infty} \gamma = 1.$$

The authors compared the calculated values of ρ and γ with available published data for the case of n-heptane. The comparison showed that the assumption of γ tending to 1 with increase in pressure leads to fairly good agreement between the results of acoustic and PVT measurements.

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Kursk State Pedagogic Institute
Original article submitted June 17, 1972

THE ZERO STATIC-PRESSURE LEVEL IN CYCLONE CHAMBERS

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UDC 533.601.1

The position of the zero excess-pressure level is important in calculating the pressure distribution in a spiral flow and also in determining the hydraulic resistance of a cyclone device. It is shown that experiment does not confirm the assumption that the zero level coincides with the radius $\eta = 1$, where the tangential velocity component is largest. A qualitative analogy exists between the pressure and tangential-velocity distributions in a circular eddy and in a real cyclone flow, and it is therefore possible to find the zero-pressure level $\eta_{\Delta\bar{P}=0}$ from the zero value for the second derivative of the pressure with respect to radius.

An approximation is used for the tangential velocity with this condition to get relationships for the dimensionless pressure:

$$\Delta\bar{P} = \frac{\Delta P}{\frac{1}{2}\rho w^2_{\varphi\max}} = 4^n \sum_{m=0}^{\infty} (-1)^{m+1} \frac{(n-m)!}{m!\,(n+m)\,(1+\eta^2)^{n+m}} + C, \tag{1}$$

$$\eta_{\Delta\bar{P}=0} = \sqrt{\frac{2n-1}{2n+1}}. \tag{2}$$

We see from (2) that the radius $\eta_{\Delta\bar{P}=0}$ can vary from 1 (for $n = \infty$) to zero (for $n = 0.5$) in relation to the flow generation conditions, which are characterized by n.

Experimental results are given on the zero-pressure level in cyclones with narrowing factors at the outlet from 0.05 to 1; calculations from (1) and (2) are used with experimental data to show that the pressure reduction at the axis increases as the relative narrowing is reduced, reaching a maximum and then decreasing rapidly, while $\eta_{\Delta\bar{P}=0}$ moves towards the flow axis until the zone of reduced pressure vanishes completely.

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Original article submitted June 26, 1972.

DETERMINATION OF THE FIRST ROOT OF THE CHARACTERISTIC EQUATIONS IN THE ANALYTICAL THEORY OF THERMAL CONDUCTIVITY

A. A. Shmukin

UDC 536.21

The characteristic equations of the analytical theory of thermal conduction are put as the series

$$f(\mu) = \sum_{n=0}^{\infty} \varphi_n \mu^{2n}, \quad \varphi_0 = 0, \tag{1}$$

where the series of coefficients $\{\varphi_n\}$ takes a particular form for each detailed problem; to determine the square of the root least in modulus for (1) one has the formula

$$\mu_1^2 = \lim_{n \to \infty} \frac{\Omega_n}{\Omega_{n+1}}, \tag{2}$$

where Ω_n is given by the recurrence relation

$$\Omega_0 = \frac{1}{\varphi_0},$$

$$\Omega_n = -\sum_{s=1}^{n} \Omega_{n-s} \frac{\varphi_s}{\varphi_0} \tag{3}$$

and these are the coefficients to μ^{2n} in the expansion of $1/f(\mu)$ as a power series in degrees of μ^2.

Equation (2) gives μ_1^2 for $n \to \infty$, but analysis indicates that the convergence to the exact value of μ_1^2 is rapid, so (2) can be put as

$$\mu_{1,n}^2 = \frac{\Omega_n}{\Omega_{n+1}}, \quad n = 0, 1, \ldots, m, \tag{4}$$

where m is a finite number, to find μ_1^2 with any preset accuracy ε when

$$|\mu_{1,m}^2 - \mu_{1,m-1}^2| \leqslant \varepsilon; \tag{5}$$

and secondly one can obtain simple formulas to give μ_1^2 approximately, i.e.,

$$\mu_1^2 \approx \frac{\Omega_0}{\Omega_1} = -\frac{\varphi_0}{\varphi_1}, \tag{6}$$

$$\mu_1^2 \approx \frac{\Omega_1}{\Omega_2} = \frac{\varphi_0 \varphi_1}{\varphi_0 \varphi_2 - \varphi_1^2}. \tag{7}$$

$$\mu_1^2 \approx \frac{\Omega_2}{\Omega_3} = \frac{\varphi_0 (\varphi_1^2 - \varphi_0 \varphi_2)}{\varphi_0 (\varphi_2 \varphi_1 - \varphi_3 \varphi_0) - \varphi_1 (\varphi_1^2 - \varphi_2 \varphi_0)}. \tag{8}$$

As (6)-(8) contain the coefficients φ_n representing the form of the characteristic equations as parameters, we clearly have (6)-(8) as universal; it is also found that (7) and (8) are highly accurate.

The algorithm of (4) and (5) goes with formulas (7) and (8) in illustrations for detailed characteristic equations from the theory of thermal conduction and mathematical physics.

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SOLUTION OF A MIXED BOUNDARY-VALUE PROBLEM FOR THERMAL CONDUCTION IN A CYLINDRICAL REGION WITH AN INCLUSION

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A particular solution is given for steady-state heat conduction in a cylinder of height $\delta_1+\delta_2$ and radius R containing an inclusion of another material of height δ_2 and radius r_1. A constant temperature is maintained at the boundary $z' = -\delta_1$, while at the boundary $z' = \delta_2$ there is heat transfer with a medium of constant temperature, and at boundary $r' = R$ there is thermal insulation.

An additional boundary condition is introduced following from simulation of the temperature distribution with the MSM-1 analog system. It is assumed that the temperature distribution over the cylindrical surface $r' = r_1$ in the main material can be considered as linear with a coefficient of proportionality b_1, while along the boundary of the two materials it is the same with a coefficient of proportionality b_2.

The problem is handled by dividing the body into four regions D_1 $(0 \le r \le 1,\ 0 \le z \le 1)$, D_2 $(1 \le r \le m,\ 0 \le z \le 1)$, D_3 $(0 \le r \le 1,\ -p \le z \le 0)$, D_4 $(1 \le r \le m,\ -p \le z \le 0)$ and solving for each region separately; as the boundary condition at z = 0 is not given for each region, the solutions contain constants of integration, which are found from the conditions for ideal thermal contact at z = 0.

The solutions are found by applying an integral Hankel transformation for the regions D_1 and D_3 of the form

$$H_i[T_i(r,\ z)] = \int_0^1 rJ_0(\nu r)\, T_i(r,\ z)\, dr \quad (i = 1,\ 3), \tag{1}$$

where ν are the positive roots of $J_0(\nu) = 0$, while the relation for the regions D_2 and D_4 is of the form

$$H_i[T_i(r,\ z)] = \int_1^m rB_0(\mu r)\, T_i(r,\ z)\, dr \quad (i = 2,\ 4), \tag{2}$$

where

$$B_0(\mu r) = J_0(\mu r) N_0(\mu) - N_0(\mu r) J_0(\mu),$$

and μ are the positive roots of

$$J_1(\mu m) N_0(\mu) - N_1(\mu m) J_0(\mu) = 0.$$

The inverse transformations take the form

$$T_i(r,\ z) = 2\sum_{n=1}^{\infty} \frac{J_0(\nu_n r)}{J_1^2(\nu_n)} H_i \quad (i = 1,\ 3), \tag{3}$$

$$T_i(r,\ z) = \frac{\pi^2}{2}\sum_{n=1}^{\infty} \frac{\mu_n^2 J_1^2(\mu_n m)\, B_0(\mu_n r)}{J_0^2(\mu_n) - J_1^2(\mu_n m)} H_i \quad (i = 2,\ 4). \tag{4}$$

The coefficients of proportionality c_1 and c_2 are found from the conditions for the mean heat fluxes through the surface r = 1 to be equal:

$$\int_{-p}^{0} \left(\frac{\partial T_3}{\partial r}\right)_{r=1} dz = \int_{-p}^{0} \left(\frac{\partial T_4}{\partial r}\right)_{r=1} dz, \tag{5}$$

$$\int_0^1 \left(\frac{\partial T_1}{\partial r}\right)_{r=1} dz = k\int_0^1 \left(\frac{\partial T_2}{\partial r}\right)_{r=1} dz. \tag{6}$$

Formulas are derived for temperature and heat flux; a cylindrical inclusion is also envisaged.

NOTATION

t, temperature of inclusion or main material; t_e, environmental temperature; t_h temperature at heated surface; $T = (t - t_e)/(t_h - t_e)$, dimensionless temperature; $\lambda_1(\lambda_2)$, thermal conductivity of inclusion (main material); $r = r'/r_1$, $z = z'/\delta_2$, dimensionless coordinates; $k = \lambda_2/\lambda_1$, $p = \delta_1/\delta_2$, $m = R/r_1$, $c_1 = b_1\delta_1 t_h/(t_h - t_e)$, $c_2 = b_2\delta_2 t_h/(t_h - t_e)$, dimensionless quantities, J_0 and J_1 Bessel functions, and N_0 and N_1 Neuman functions.

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TRENDS IN CONSTANT-TEMPERATURE LINE MOTION IN SOLIDS

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Nonstationary thermal conduction is considered as serial displacements of the isotherms Θ = idem; this naturally introduces the paths traversed by the fronts Θ = idem and the velocities v_Θ; the analysis shows that the speed of Θ = idem in a half-space with Bi = ∞ is

$$v_\Theta = \left(\frac{\partial x}{\partial \tau}\right)_\Theta = \frac{2a\Phi^2(\Theta)}{x}, \quad \Phi(\Theta) \text{ — reciprocal erfc,} \tag{1}$$

i.e., is dependent on the path traveled x and on Θ; the shape of the Θ = idem curves in x and τ coordinates corresponds to the upper branch of a quadratic parabola.

On the other hand, for bounded bodies in the regular thermal state we get the dimensionless velocity $\bar{v}_\Theta = [\partial(1-\eta)/\partial\,\mathrm{Fo}]_\Theta$ for a plate, cylinder, and sphere, respectively, for boundary conditions of the first and third kinds as

$$\mu_1 \operatorname{ctg} \mu_1\eta; \quad \mu_1 \frac{J_0(\mu_1\eta)}{J_1(\mu_1\eta)}; \quad \frac{\mu_1^2}{\dfrac{1}{\eta} - \mu_1 \operatorname{ctg} \mu_1\eta}, \tag{2}$$

where $\mu_1 = \mu_1(\mathrm{Bi})$, and $\eta = x/l_0$.

Then the regular condition is represented by an unaltered velocity $\bar{v}_\Theta$ specific for each point, which is independent of Θ. Minsk-22 calculations show that this corresponds to equidistant parts of the curves for the various fronts Θ = idem in $(1-\eta)$–Fo coordinates (Fig. 1). The speed $\bar{v}_\Theta$ in a plate, cylinder, or sphere may be taken as in the ratio 1:2:3 for the points η = idem.

In the quasistationary state with boundary conditions of a second kind we have

$$v_\Theta = \left[\frac{\partial(l_0 - x)}{\partial\tau}\right]_\Theta = \frac{ma}{x} \quad \text{and} \quad \bar{v}_\Theta = \left[\frac{\partial(1-\eta)}{\partial\,\mathrm{Fo}}\right]_\Theta = \frac{m}{\eta} \tag{3}$$

where m = 1, 2, and 3 for plate, cylinder, and sphere respectively; here again the velocity $v_\Theta(\bar{v}_\Theta)$ is unvarying at each point in the body.

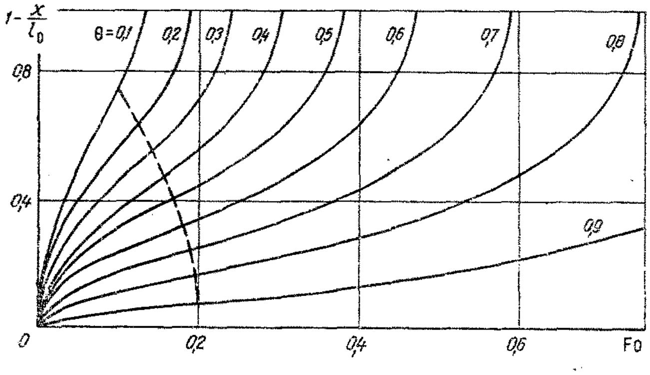

Fig. 1. Course of fronts Θ = idem in an unbounded plate for Bi = ∞.

This relationship applies for bounded bodies when there are continuous uniformly distributed heat sources and symmetrically disposed instantaneous source; the $\bar{v}_\Theta$ are determined by (2) and are not dependent on the outputs of the heat sources nor on the disposition of the instantaneous source in the body.

This provides a general feature for regularization in the thermal kinetics: the $\bar{v}_\Theta$ for the isotherms in a bounded body are unaltered at each point in the regular and quasistationary states and are dependent only on the coordinates of the point and the nature of the boundary conditions.

The initial stage preceding the regular one may be examined by comparing the behavior of the fronts Θ = idem in bounded bodies and in a half-space; in comparison with the latter, the standard solutions for the temperature distribution for $\mathrm{Bi} = \infty$ must be interpreted in the form of the course of Θ = idem in $(1-\eta)-\sqrt{\mathrm{Fo}}$ coordinates, while for $\mathrm{Bi} \neq \infty$ one must take the $\mathrm{Bi}(1-\eta)-\mathrm{Bi}\sqrt{\mathrm{Fo}}$ coordinates. Calculations show that the initial parts of the lines Θ = idem for the plate up to the regular step correspond to the thermal laws for the half-space with identical boundary conditions. The trend is also derived for a cylinder and sphere as the fronts Θ = idem recede from the outer surface, where deviations occur on account of geometrical changes in the fronts.

In any case one can say that the peripheral layers of a bounded body are subject to the thermal laws applicable to a half-space, while the central layers are involved in the steady-state stage, and are characterized by unvarying local speed in the isotherm motion. Meanwhile, the geometrical region to which this law applies expands.

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REGULARIZATION OF THE THERMAL CONDITIONS IN THE CENTER OF A BOUNDED BODY

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It has previously been shown that the speeds $v_\Theta = [\partial(l_0 - x)/\partial\tau]_\Theta$ of the isotherms Θ = idem in the regular state are as follows for a plate, cylinder, and sphere subject to boundary conditions of the first and third kinds:

$$\frac{a}{l_0}\mu_1 \operatorname{ctg}\mu_1\eta;\quad \frac{a}{l_0}\mu_1\frac{J_0(\mu_1\eta)}{J_1(\mu_1\eta)};\quad \frac{a}{l_0}\frac{\mu_1^2}{\frac{1}{\eta}-\mu_1\operatorname{ctg}\mu_1\eta}. \tag{1}$$

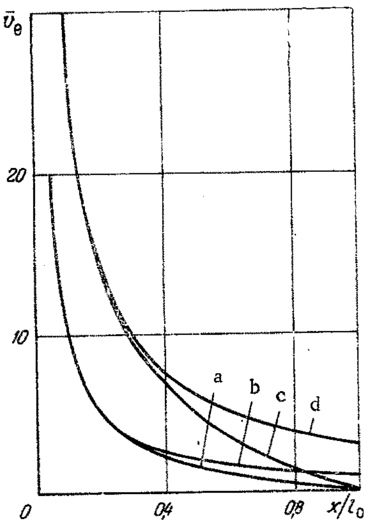

Fig. 1. Relation of $\bar{v}_\Theta$ to η for an unbounded plate (a, $\mathrm{Bi} = \infty$, b, $\mathrm{Bi} = 0.01$) and sphere (c, $\mathrm{Bi} = \infty$; d, $\mathrm{Bi} = 0.01$).

Computer calculations (Fig. 1) show that the conditions a = idem, l_0 = idem, Bi = idem, $\eta = x/l_0$ = idem cause the v_Θ to be in the ratio 1:2:3 in these bodies, i.e., as the ratio of the volumes of the bodies to the areas of the bounding surfaces.

If $\mu_1\eta$ is small, it is shown analytically that the velocity $\bar{v}_\Theta = v_\Theta l_0/a$ is independent of the thermal setting at the outer boundary of the body, since in that case we have

$$\bar{v}_\Theta = \frac{m}{\eta}, \tag{2}$$

where m = 1, 2, 3 for plate, cylinder, and sphere, respectively.

Formula (2) applies for any Bi in the central parts of a body $0 \le \eta \lesssim 0.25$, and to the whole of the body for $\mathrm{Bi} \le 0.1$.

It is shown that (2) allows one to determine the thermal diffusivity a without reference to the thermal conditions at the outer boundary.

If there is a simple exponential relationship between Θ and Fo, (2) indicates an elliptical temperature distribution for the temperature which for a given Fo takes the form

$$\Theta = \text{const}_m(\mu_1, \text{Fo}) \sqrt{1 - \frac{\mu_1^2}{m}\eta^2}. \quad (3)$$

Then the elliptical distribution of Θ in η applies for any Bi in the thermal condition for the central part of the body $0 \le \eta \lesssim 0.25$, while it applies to the whole body for Bi ≤ 0.1.

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HEAT TRANSFER IN A SYSTEM OF TWO SEMI-INFINITF BODIES SEPARATED BY AN INTERLAYER

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UDC 621.73.043

The actual heat transfer between two solids often occurs via a layer whose properties substantially influence the temperature distribution and heat fluxes.

For instance, the heat transfer between tool and workpiece in hot pressure working of metals commonly occurs through a layer of scale, with lubrication by gas and other inclusions [1].

Standard solutions [2, 3] provide the current parameters of this layer on the hot and cold sides, and then the solution for a semi-infinite body with boundary conditions of the first kind [4] is used to find the temperature distributions in the bodies. The heat transfer in a system of two massive bodies indicates that the solutions of [2, 3], which were derived on the assumption of zero specific heat in the layer, are inapplicable for brief contact and when the layer is of considerable thickness. In that case, the heat transfer is substantially affected by the layer heat capacity, and under certain conditions the third body may be completely insulated if the contact time τ and the layer thickness δ are appropriate. The solution for ideal contact between two semi-infinite bodies [4] implies that to within about 2.5% The third body is thermally insulated for $\text{Fo} = a\tau/\delta^2 \le 0.1$ (a is the thermal diffusivity of the layer); the third body is involved in the heat transfer if $\text{Fo} > 0.1$.

To determine the temperature distribution in each of the three bodies as a function of time, a study has been made of the triple heat transfer using a model of two massive bodies separated by a layer.

The dimensionless temperatures of the contact surfaces have been calculated for set values of Fo, using similarity theory, and graphs have been constructed for the variations in the temperature, which are given by analytical expressions for the temperature distribution in each body and the heat loss in the layer in relation to the thermophysical and geometric parameters of the latter. The solutions are suitable in particular for calculating thermal processes in pressure working of metals.

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